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# FRICTION AND HEAT TRANSFER IN THE SWIRL FLOW OF WATER IN AN ANNULUS

**R. A. SEBAN and A. HUNSBEDT***

Department of Mechanical Engineering, University of California, Berkeley, California, U.S.A.

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**Abstract**—Measurements of friction and of heat transfer on an annulus in which swirl flow of water was produced by a single helical vane indicate that the friction and the heat-transfer coefficients for turbulent flow can be predicted by the appropriate adjustment of available values for straight channels. Friction in the laminar range exhibits similarity with that found for curved tubes but the interpretation of the few heat transfer results obtained for this region is clouded by the possible contributions of free convection.

## INTRODUCTION

THIS paper presents experimental results for the friction and heat transfer in an annulus containing a helical vane to produce spiral flow of the water flowing through the annulus. Gutstein *et al.* [1] have given results for such systems with air flow, and for the associated problem of a tube containing a twisted tape, Lopina and Bergles [2] have presented results for turbulent flow and have defined most of the current status of this spiral flow problem in the turbulent regime.

For turbulent flow, the usual consideration of this helical flow is one of "straightening out" the helical channel and applying to it the accepted formulations for flow in a straight channel. Gutstein [1] shows that the friction and the heat transfer can be predicted reasonably in this way. Lopina and Bergles [2] show that the heat transfer can be predicted in this way and indicate that for a complete appraisal a free convection effect should be considered when the temperature of the outer wall exceeds that of the fluid.

The present results support these views for turbulent flow, by means of friction data for axial Reynolds numbers from 1200 to 40000 and heat transfer data for axial Reynolds numbers from 2300 to 20000, and the presentation of the results enables some further comments upon the methods for the prediction of the friction and the heat transfer for the spiral flow in an annulus produced by a single helical vane.

## FRICTION EXPERIMENTS

Results for friction for the isothermal flow of water were obtained for the annulus between 48 in. (122 cm) long concentric Plexiglas tubes, with inner and outer diameters of 0·625 in (1·59 cm) and 1·125 in. (2·85 cm). Pressure taps were located 9 in. (22·4 cm) from the ends and at the mid length of the annulus.

The helical "vane" was a 0·31 in. (0·79 cm) o.d. rubber tube which was wound on the complete length of the inner tube with the desired pitch, and inflated after insertion into the outer tube. The pitches were chosen so that the pressure taps were located 180° from the vane. Ratios of pitch, $P$ to outer annulus diameter $D_0$, ranged from 1·78 to 8·90.

Comparisons of the upstream and downstream pressure drop values revealed no differences which would have implied an entrance effect, and the downstream drop was used to evaluate an

* Present address: General Electric Company, Sunnyvale, California.

apparent friction coefficient on the basis of the definition:

$$f_A = \frac{\Delta p}{\Delta x}(D_H)\frac{2}{\rho u_x^2}. \qquad (1)$$

Here $\Delta x$ is the axial distance between the pressure taps, $u_x$ is the mean axial velocity based on the open area of the cross section perpendicular to the axis, obtained by considering for the cross section of the tube a trapezoid having an area equal to the original cross section of the tube, and $D_H$ is the hydraulic diameter for the net axial flow area.

The apparent friction factors as obtained from this test sections are shown on Fig. 1, for four

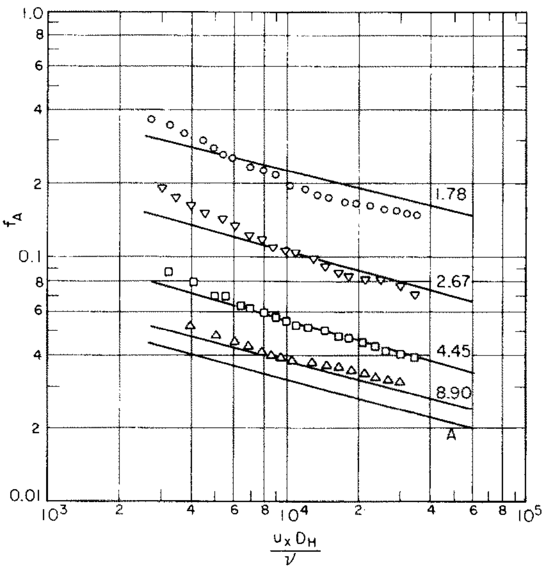


FIG. 1. Apparent friction factors from the Plexiglas system. Line A is equation (2). The other lines are shifted values of equation (2) for the pitch to outside diameter ratios as indicated.

pitch ratios, as a function of the axial Reynolds number, $u_x D_H/\nu$.

Results for friction for isothermal and non-isothermal conditions were also obtained for the annulus between a 0·509 in. (1·29 cm) i.d. type 3·04 Stainless Steel tube and a 0·304 in. (0·77 cm)

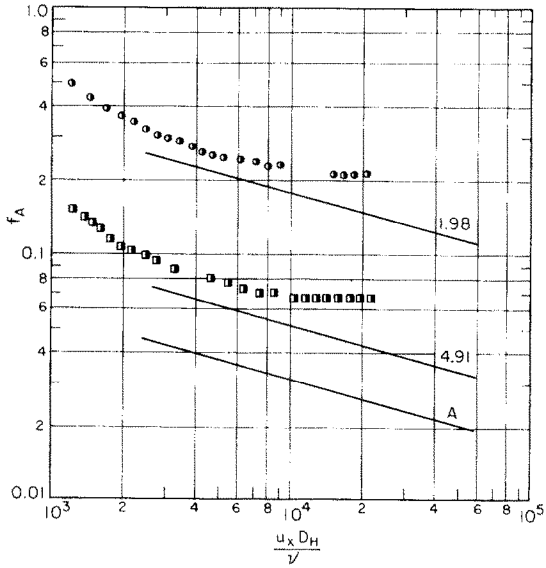


FIG. 2. Apparent friction factors for the stainless-Rulon system. Line A is equation (2). The other lines are shifted values of equation (2) for the pitch to diameter ratios as indicated.

Rulon rod, machined to this dimension from a 0·500 in. (1·27 cm) rod to leave a helical vane 0·12 in. (0·30 cm) in thickness. The overall length of the annulus was 31 in. (79 cm) and pressure taps were located 3 in. (7·65 cm) from the ends. Two inserts, having pitch to o.d. ratios of 4·91 and 1·98 were used, these being such that the pressure taps were located 180° from the vane.

Apparent friction factors, for isothermal conditions, are shown on Fig. 2. Non-isothermal results, obtained when the water was heated by the outer tube, agree with these values when properties are evaluated at the film temperature at the mid length of the heated section.

## PREDICTION OF THE FRICTION

The appraisal of the friction is based on the view that the helical flow is the same as that in a straight channel, of the same hydraulic diameter and a length equal to the helix length along the outside diameter. This length estimate is arbitrary, because any length down to the shortest one, the helix length along the inside diameter, could be used. Thus if $D$ is the appropriate diameter, the helix length is $x/\cos\alpha$, where $\alpha = \tan^{-1} \pi D/P$, and the total Reynolds number is $u_T D_H/\nu$, where $u_T = u_x/\cos\alpha$ and $D_H$ is retained as in equation (1) because of the difficulty in defining the cross section normal to the helical flow (the difference between this $D_H$ and the true $D_H$ is not very great). The apparent friction coefficient $f_A$ is then $f_T/\cos^3\alpha$ where $f_T$ is the coefficient for the channel, which for turbulent flow is taken to be given by the Blasius equation

$$f_T = 0{\cdot}316\left(\frac{\nu}{u_T D_H}\right)^{0\cdot25} \tag{2}$$

The curves on Figs. 1 and 2 indicate the values of $f_A$ obtained in this way with $D = D_0$. On Fig. 1, the predictions are within 15 per cent of almost all of the data, but the form of the data for the smaller pitches is not that of the prediction. The nature of the correspondence is essentially that found by Gutstein *et al.* [1] and it is notable also that a prediction like this will serve about as well as for the twisted tape results of [2].

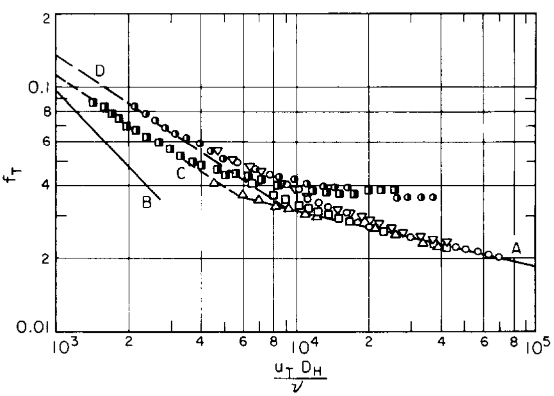


FIG. 3. True friction factors. Line A is equation (2), line B is the factor for laminar flow between parallel walls, line C and D are adjustments of line B for curvature. Point designations are the same as Figs. 1 and 2 and the applied values of cos $\alpha$ are given in Table 1.

The prediction on Fig. 2 does not show this kind of correspondence and, except in the intermediate Reynolds numbers, there is substantial departure from the prediction, with the relatively constant friction factors at high Reynolds numbers implying some roughness effect. It must be pointed out that in the heat-transfer system there existed a radial clearance of 0·004 in. (0·01 cm) between the vane and the tube and that leakage could have had some effect, but this would be in the direction of lowering the apparent friction coefficient rather than raising it.

*Table 1. Values of cos α*

| $P/D$ | 8·90 | 4·45 | 2·67 | 1·78 | 4·91 | 1·98 |
|---|---|---|---|---|---|---|
| Results from Fig. | 1 | 1 | 1 | 1 | 2 | 2 |
| cos $\alpha$, Fig. 3 | 0·91 | 0·82 | 0·66 | 0·52 | 0·84 | 0·54 |
| cos $\alpha$, on $D_0$ | 0·94 | 0·82 | 0·65 | 0·49 | 0·84 | 0·53 |
| cos $\alpha$, on $D$ mean | 0·96 | 0·88 | 0·73 | 0·58 | 0·89 | 0·62 |

Another view of the friction factor situation is obtained by representing $f_T = f_T(u_T D_H/\nu)$, as in Fig. 3, and forcing the results of Fig. 1 to fit the Blasius equation at high Reynolds numbers by appropriately selecting the value of cos $\alpha$. Table 1 contains these values, and shows as well the values of cos $\alpha$ based on the outer and on the average of the inner and outer diameters of the annulus.

The points from Fig. 1 now show a departure from equation (2) at the low Reynolds numbers, depending in amount inversely on the pitch ratio. The points from Fig. 2 appear also, but these are oriented to provide approximately a continuation to low Reynolds numbers of the results from Fig. 1 for about the same pitch ratio and to produce similar friction factors for the two pitch ratios at the highest Reynolds numbers, in the view that the relatively constant friction factors that occur there reveal a roughness effect. The values of cos $\alpha$ required for this shift of the values from Fig. 2 are contained in Table 1 and they are close to the values associated with the outside diameter of the annulus.

Curve B of Fig. 3 shows the friction factor for laminar flow between parallel walls, and for the radius ratio involved this is a fair approximation for an annulus ([37], p. 59). Despite the fact that dye injection experiments on the Plexiglas test section failed to reveal any secondary flow pattern at any Reynolds number, recourse here is made arbitrarily to the results for curved pipes, for which in the laminar regime ([4], p. 530)

$$\frac{f}{f_0} = 0{\cdot}28\left[\frac{uD_H}{\nu}\bigg/\sqrt{\left(\frac{R}{r_c}\right)}\right]^{0\cdot36} \tag{3}$$

where $R$ is the tube radius and $r_c$ the radius of curvature of the pipe. Taking $R$ as the hydraulic radius of the annulus and an approximation for the curvature of the helix as

$$\frac{R}{r_c} \approx \frac{1 - D_i/D_0}{(P/2D)^2 + 2}$$

then the use of equation (3) and Line B gives $f \sim (u_T D_H/\nu)^{-0\cdot64}$ and the lines C and D of Fig. 3 correspond to equation (3) with line B and an additional factor of 0·60. The actual friction factors are above the indications of lines C and D in the region of the intersection of these lines with line A, so that the occurrence of "fully turbulent" flow is implied to occur at Reynolds numbers well beyond this intersection. For $P/D = 1{\cdot}78$ the actual attainment of a fully turbulent condition is indicated at a Reynolds number of about $2{\cdot}5 \times 10^4$. This is not inconsistent with what a similar interpretation would indicate from the results of Gutstein [1].

In respect to this orientation of the low Reynolds number results on Fig. 3 it is significant to note ([5], p. 322) that laminar flow in a curved channel is unstable at Reynolds numbers above $u_T D_H/\nu = 50(r_c/R)^{\frac{1}{2}}$, and this indicates Reynolds numbers of 140 for the small and 230 for the large pitch. The consequence of the resulting three dimensional motion is a gradual transition, like that which exists for the curved pipe. It is a view like this that is needed to account for the magnitude of the friction factors, for the results of Cheng and Akiyama [6], for curved rectangular channels, which extend only to an aspect ratio of 5, imply a relatively small effect of curvature on friction for the larger aspect ratios of the present experiments.

The representation used on Fig. 3 requires a roughness effect to rationalize the magnitude and behaviour at high Reynolds numbers of the friction factors from the steel tube system, and comparison with the data for sand roughness in pipes ([4], p. 521) requires a ratio $D_H/4\varepsilon = 46$, which in terms of the annulus hydraulic radius implies $\varepsilon \approx 0{\cdot}001$ in. (0·0025 cm). Measurements on the machined surface of the Rulon insert indicated an r.m.s. roughness of 0·002 in. (0·005 cm) and this, in conjunction with the far smoother tube, rationalizes the magntidue of the roughness estimated from the data.

## HEAT TRANSFER EXPERIMENTS

The steel tube was fitted with bus bars to give a 21 in. (53 cm) heated length between them, heating being provided by electrical dissipation from the a.c. current flow in the 0·058 in. (0·147 cm) thick tube wall. Thermocouples provided for the measurement of inlet and outlet mixed mean temperatures and of the temperature of outside of the tube wall, where the couples were taped directly on to the wall (equivalent results were obtained when the thermocouple was electrically insulated from the wall). Eight wall temperature thermocouples were located along the pipe, all 180° from the vane, with five additional circumferential couples located at the midpoint of the heated length. The tube exterior was insulated with a layer of fiberglass insulation. Balances between the power input and the increased energy of the water stream were within 4 per cent and the higher power input was used for the evaluation of the heat-transfer coefficient.

The wall temperatures in the downstream half of the heated section were generally parallel to the linear variation of mixed mean temperature consequent upon the uniform heat input. At low Reynolds numbers with the $P/D = 4{\cdot}91$ pitch ratio there was in the upstream region departure from the linear downstream variation. This was not in the nature of higher heat transfer coefficients, as for a thermal entry effect, but partly periodic, reminiscent of what was found with laminar flow in coiled tubes ([7], p. 390). The effect did not occur with $P/D$ = 1·98, where the linear relation was maintained to within 2 pitch lengths of the point where heating began ([7], p. 390).

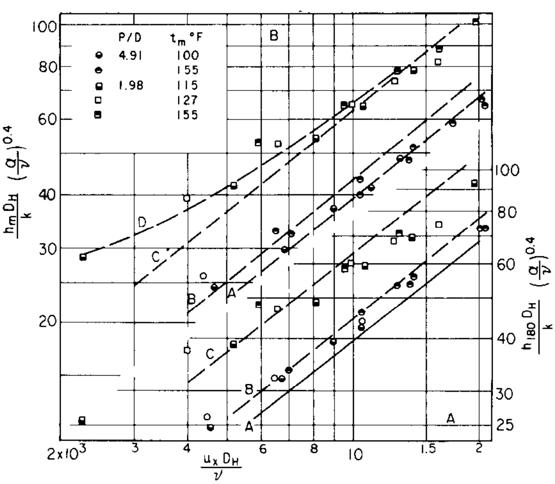


FIG. 4. Heat transfer coefficients. Line A is the prediction for turbulent flow between parallel plates heated on one side. Lines B and C are shifts of line A using the same cos $\alpha$ as on Fig. 3. Approximate water temperatures at the mid length of the test section are indicated. Curve D is equation 5.

Inside tube wall temperatures were predicted on the basis of uniform heat generation and no circumferential heat flow, and the tube wall was thin enough to make this plausible except in the immediate region of the vane. Thus, away from the vane, the heat flux was inferred from the heat generation assuming no peripheral heat conduction, and the heat transfer coefficient could then be evaluated. Figure 4(a) shows these values for the 180° location, incorporated into the usual groups, with properties evaluated at the film temperature at the midpoint of the heated

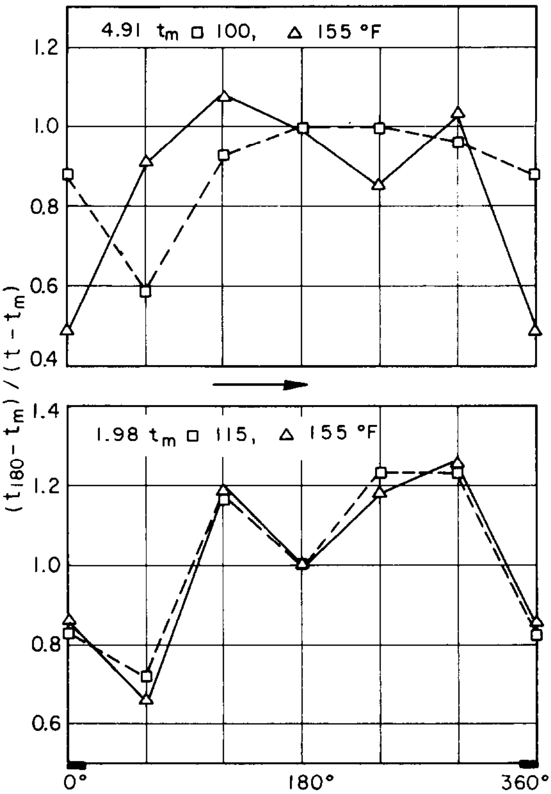


FIG. 5. Circumferential variation of calculated inner tube temperatures. The inner wall temperature is $t$ and the water temperature is $t_m$. The lines serve only to orient the points. The solid sections on the abscissa indicate the vane half width, and the twist of the channel is right handed into the page.

length. The abscissa is the axial Reynolds number, as used on Fig. 2.

The considerable circumferential variation of wall temperature that occurred is illustrated by Fig. 5, which shows typical distributions of the inside wall temperatures, evaluated as just indicated, by the group $(t_{180} - t_m)/(t - t_m)$, which would be $h/h_{180}$ if the heat flux were indeed peripherally uniform. The circumferential temperature variation in the region of the vane is great enough to produce a peripherally non uniform heat flux, and analyses of the effect is complicated by the geometry of the system, by the effect of leakage over the vane, and by the occurrence in most cases of the minimum temperature near but not at the vane location.

It is the uncertainty about the heat flux that impairs the evaluation of a heat transfer coefficient based on the peripheral average temperature. If the vane area is subtracted from the inside tube area and all of the heat transfer charged against the remaining surface, the transfer coefficients as shown on Fig. 4A will be increased by the factors 1·09 and 1·15 for the large and small pitches, respectively. If the average inner wall temperature difference (the mean ordinate of Fig. 5) is used for the evaluation of the coefficient, the coefficient will be reduced by a factor which is the mean ordinate. This factor varies; for the larger pitch it ranges from about 0·88 at low Reynolds number to 0·82 at high and for the small pitch from 0·98 to 0·96. Thus the heat transfer coefficients, $h_m$, based on the mean temperature, and excluding the area covered by the vane, tend to be lower than $h_{180}$ for the large pitch and higher for the small one. These coefficients are shown on Fig. 4(b), where it is evident that the introduction of the factors for the mean ordinate from distributions like those shown on Fig. 5 has somewhat increased the scatter of the results.

#### PREDICTION OF THE HEAT TRANSFER

In correspondence with the view adopted for the prediction of the friction, the forecast of the heat transfer for turbulent flow is made from the available theoretical values for flow between parallel plates heated on one side and insulated on the other ([3], p. 180). Curve A of Fig. 4 represents the heat transfer coefficient for this situation and this is construed to specify for the helical vane system the heat-transfer coefficient in terms of the total Reynolds number $u_T D_H/\nu$. To obtain the coefficient for the vane system in terms of the axial Reynolds number $u_x D_H/\nu$ curve A is shifted to the left by the factor $u_x/u_T$, which is the value of cos $\alpha$ used in conjunction with Fig. 3. This produces curves B and C, for the two pitches for which the heat transfer was determined. At the higher Reynolds number, $h_{180}$ for the larger pitch is fairly well predicted by Curve B, while $h_m$ is nearer the straight channel performance. For the 1·98 pitch ratio Curve C predicts $h_m$ somewhat better than it does $h_{180}$. Any correspondence in this region implies, of course, that the friction at the outer, heated, surface is close to that of equation (2), and that the higher friction coefficients that are indicated on Fig. 3 are due to the roughness on the Rulon surface, as already considered.

In the view that the friction coefficients on the heat-transfer surface are more appropriately given by the results for the Plexiglas system as those are shown on Fig. 3, that figure indicates a 10 per cent departure of the friction factor from equation (1) at a Reynolds number $u_T D_H/\nu$ of about 15000 for the small pitch and 6500 for the larger, corresponding to Reynolds numbers, $u_x D_H/\nu$ of 8500 and 5800 on Fig. 4. For the larger pitch no trend is apparent at Reynolds numbers lower than this and for the small pitch there appears to be a tendency for the heat transfer coefficients to exceed those of line C but this apparent tendency is partly due to the high values of the coefficient for the runs at lowest Reynolds numbers for each of the series of results that are designated separately on the figure.

Natural convection may be a factor, and Lopina and Bergles [2] showed that the addition of a free convection coefficient, calculated from the correlation for a heated plate facing upward, with the acceleration calculated from the mean circumferential velocity and the outer radius, was necessary for the rationalization of their data. The "additional" Nusselt number is about

$$\frac{h\delta}{k} = 0{\cdot}10\left[2\left(\frac{u_a D_H}{\nu}\right)^2\left(\frac{D_H}{D}\right)(\tan^2\alpha)\,\beta\Delta T\frac{\nu}{\alpha} \times\left(\frac{\delta}{D_H}\right)^3\right]^{\frac{1}{3}} \tag{4}$$

and Lopina and Bergles took $\delta = D_H$. Doing this for the small pitch with the temperature differences that existed gives "additional" values of Nusselt number of 6–10 at low Reynolds numbers and of 20 at high Reynolds numbers, and the data could accommodate such an addition to Curve C at the low but not at the high Reynolds numbers.

Equation (4) is also a reasonable approximation for cellular convection between horizontal isothermal surfaces (though at low Rayleigh numbers an alternative expression using a smaller exponent is preferable) and this system is more compatible with the visualization of additional natural convection at the outer surface of the annulus. With this system the distance $\delta$ should extend inward to the point at which the radial variation of temperature is no longer large, and with water in an annulus, this distance is very much smaller than $D_H$. This distance will decrease with Reynolds number, and Table 2 contains values of the Rayleigh number calculated with $\delta$ evaluated at $\delta(\sqrt{\tau/\rho})/\nu = 75$, and friction coefficients taken from Fig. 3 for the small pitch, the entries being for the highest and lowest Reynolds numbers for the series designated on Fig. 4 by squares shaded above and below.

*Table* 2

| Fluid temp. | $\Delta t$ | $u_x D_H/\nu$ | $(hD/k)(d/\nu)^{0\cdot4}$ | $Ra$ |
|---|---|---|---|---|
| 117 | 14 | 2300 | 28·5 | 2420 |
| 104 | 4·5 | 13900 | 79 | 125 |
| 170 | 30 | 5950 | 53 | 6520 |
| 154 | 14 | 19500 | 105 | 1200 |

The value of $\delta^+ = 75$ that is the basis of the table is arbitrary, but it corresponds roughly to the region of substantial radial temperature variation for the Prandtl numbers involved, to show the trend of Rayleigh number with operating conditions. If 1200 is chosen as the threshold of the convective motions considered, there is more of an opportunity for their occurrence at low rather than at high Reynolds numbers.

Finally it is noted that the heat-transfer coefficients on Fig. 4 can be predicted in a slightly better way by use of the expression

$$\frac{hD_h}{k} = \frac{f}{8}\frac{u_T D_H}{\nu}\left(\frac{\nu}{\alpha}\right)^{0\cdot4} \tag{5}$$

evaluating the friction $f$ from Fig. 3. This will give essentially the same prediction as Curves A and B at high Reynolds numbers, and higher predictions for the Nusselt number at the lower Reynolds number, to the degree that the friction coefficients on Fig. 3 depart from Curve A on that figure. This curve, shown as D on Fig. 4(a) is a better fit, but is not at the present time completely definitive because of the possible effects of free convection and the fundamental question of its applicability when the flow is not truly turbulent.

## CONCLUSIONS

Additional friction data have been presented for the turbulent swirl flow of water in an annulus as produced by a single helical vane. These confirm the existing view about the prediction of the friction coefficients from those of a straight channel, based on a total mean velocity and a channel length inferred from the helix angle based on the outer radius.

Limited results for low Reynolds numbers imply a laminar flow behaviour similar to that which exists for curved pipes. The attainment of a fully turbulent flow is delayed to relatively high Reynolds numbers, as it is for the curved pipe.

Results for the heat-transfer coefficient reveal a fair prediction from those for turbulent flow in a channel, though deviations exist at low Reynolds numbers which may be due to natural convection effects.

## ACKNOWLEDGEMENT

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## FROTTEMENT ET TRANSFERT THERMIQUE DANS L'ECOULEMENT TOURBILLONNAIRE D'EAU DANS UN ESPACE ANNULAIRE

**Résumé**— Des mesures de frottement et de transfert thermique pour un espace annulaire dans lequel on produit un écoulement tourbillonnaire d'eau par une ailetage hélicoïdal, montrent que les coefficients de frottement et de transfert thermique pour l'écoulement turbulent peuvent être calculés par une adaptation appropriée des valeurs connues pour les canaux rectilignes. Le frottement dans le domaine laminaire présente une similitude avec celui relatif aux tubes courbes mais l'interprétation des quelques résultats thermiques obtenus pour ce domaine est brouillée par l'intervention possible de la convection naturelle.

## REIBUNG UND WÄRMEÜBERGANG IN EINER DRALLSTRÖMUNG FÜR WASSER IM RINGSPALT

**Zusammenfassung**— Messungen der Reibung und des Wärmeübergangs an einem Ringspalt, in dem durch eine einzelne spiralförmige Rippe eine Drallströmung mit Wasser erzeugt wurde, zeigen, dass für die turbulente Strömung die Reibungs- und Wärmeübergangskoeffizienten vorausgesagt werden können durch geeignete Anpassung der verfügbaren Werte von geraden Kanälen. Die Reibung im laminaren Bereich weist Ähnlichkeit mit der in gekrümmten Rohren auf, doch war eine exakte Interpretation der wenigen Wärmeübergangswerte in diesem Bereich wegen des möglichen Einflusses der freien Konvektion nicht möglich.

## ТРЕНИЕ И ТЕПЛООБМЕН В ЗАКРУЧЕННОМ ПОТОКЕ ВОДЫ В КОЛЬЦЕВОМ КАНАЛЕ

**Аннотация**—Измерения теплообмена и трения при течении воды в кольцевом канале, в котором завихрения создавались с помощью винтообразной лопасти, показали, что коэффициенты теплообмена и трения в турбулентном потоке можно определить путем соответствуюшего пересчета значений для прямых каналов. В ламинарном режиме трение в искривленных трубах и прямых каналах аналогично, а обобщение немногочисленных данных по теплообмену в этой области затруднено из-за влияния свободной конвекции.